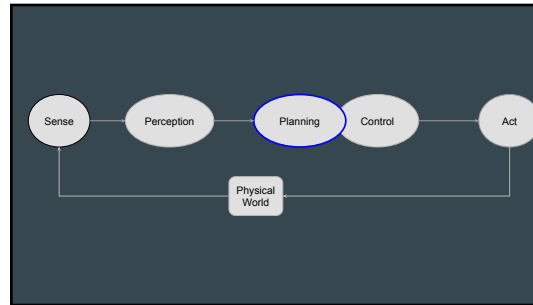


# CS4501 Robotics for Soft Eng

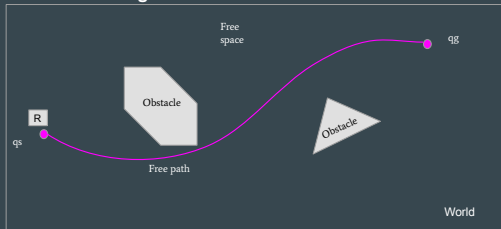
## Motion Planning II



## Motion Problem

- Given
  - World Space  $W$
  - Obstacle Regions  $O$
  - Robot State  $R$
  - Starting and Ending Configurations  $q_s, q_g$
- Find a path that modifies  $R$  so that
  - From  $q_s$  to  $q_g$
  - While staying in  $W$
  - Without hitting any obstacle  $O$
  - [other constraints]

## Motion Planning Problem



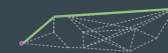
## Motion Planning Families

- Reactive
  - Bug
  - Dynamic window
  - ...
- Model-based
  - Visibility
  - Grid
  - Probabilistic
  - ...

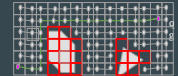
Work under different assumptions about sensor types and world models available

## Model-based Approaches Produced a Graph

Path Planning: Visibility Methods



Path Planning: Grid Methods



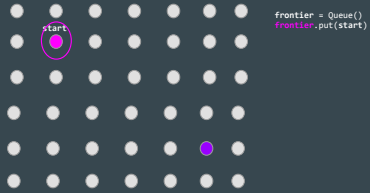
Path Planning: Probabilistic Roadmap



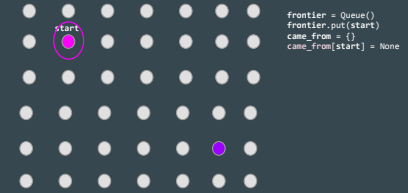
## Model-based Approaches - Searching Shortest Path in Graph

- Generic
  - BFS (Breath First)
  - DFS (Depth First)
- Informed
  - "Heuristic" to guide the search

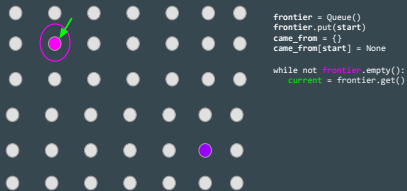
## Searching for a Path in a Graph: BFS



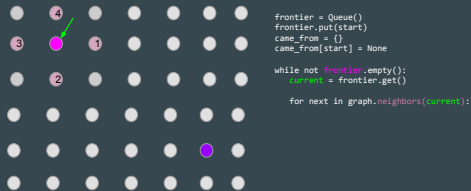
## Searching for a Path in a Graph: BFS



## Searching for a Path in a Graph: BFS



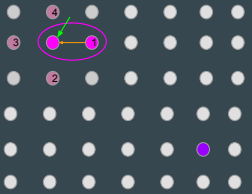
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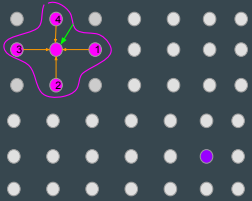


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while not frontier.empty():
    current = frontier.get()

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## Searching for a Path in a Graph: BFS

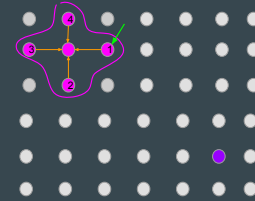


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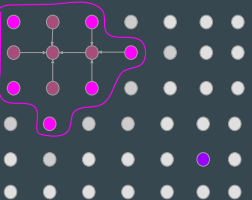


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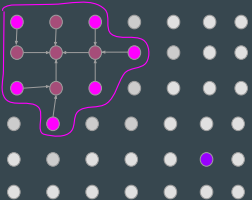


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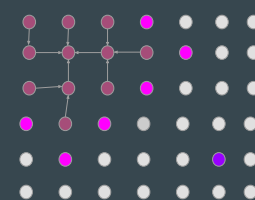


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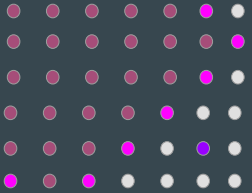


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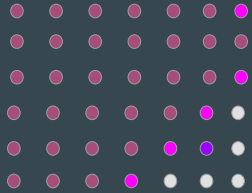


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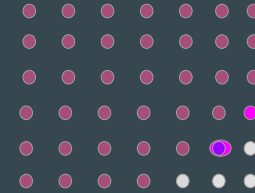


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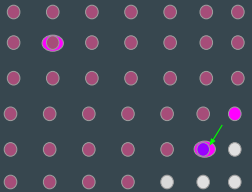
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## Searching for a Path in a Graph: BFS

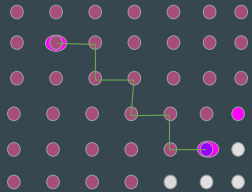


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            came_from[next] = current

path = []
while current != start:
    path.append(current)
    current = came_from[current]
path.append(start)
path.reverse()
```

## Searching for a Path in a Graph: BFS

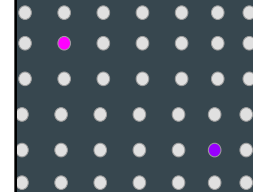


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## Searching for a Path in a Graph: Dijkstra



### Searching for a Path in a Graph: Dijkstra

- Edges with different costs
  - Very slow roads (x10 worse)
  - Diagonal are more expensive
  - Going close to obstacles more risky

### Searching for a Path in a Graph: Dijkstra

- Edges with different costs
  - Very slow roads (x10 worse)
  - Diagonal are more expensive
  - Going close to obstacles more risky
- Changes frontier exploration
  - Track costs with priority queue (return low-cost first)
  - Add a path only if it is better than best previous path
- Slightly more expensive than BFS
  - $O(V+E)$  vs  $O(V+E \cdot \log(V))$

### Searching for a Path in a Graph: Dijkstra

```

frontier = PriorityQueue()
frontier.put(start, 0)
came_from = {}

came_from[start] = None

while not frontier.empty():
    current = frontier.get()
    if current == goal:
        break
    for next in graph.neighbors(current):

```

*Low cost first*

### Searching for a Path in a Graph: Dijkstra

```

frontier = PriorityQueue()
frontier.put(start, 0)
came_from = {}
cost_so_far = {}
came_from[start] = None
cost_so_far[start] = 0

while not frontier.empty():
    current = frontier.get()
    if current == goal:
        break
    for next in graph.neighbors(current):
        new_cost = cost_so_far[current] + graph.cost(current, next)

```

*Low cost first*

### Searching for a Path in a Graph: Dijkstra

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    if current == goal:
        break
    for next in graph.neighbors(current):
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        cost_so_far[next] = new_cost
        priority = new_cost
        frontier.put(next, priority)
        came_from[next] = current

```

*Low cost first*

*Add to frontier only if it is better than best path to next*

### Searching for a Path in a Graph: Dijkstra

```

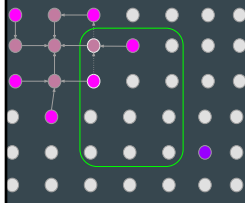
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```

*Low cost first*

## Searching for a Path in a Graph: Dijkstra

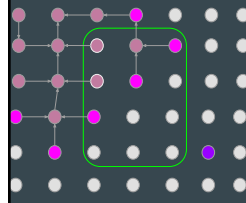


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## Searching for a Path in a Graph: Dijkstra

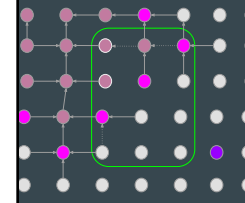


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## Searching for a Path in a Graph: Dijkstra

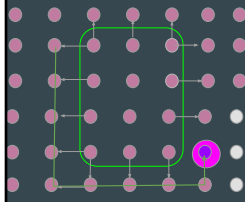


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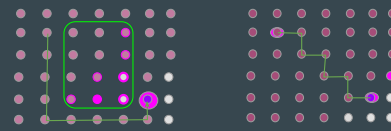


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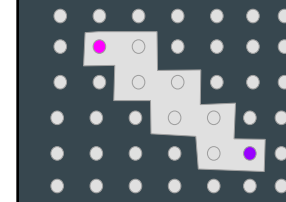
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## Dijkstra vs Breadth-First-Search



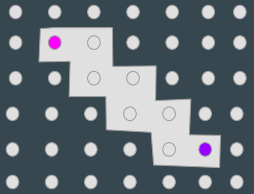
- Both find shortest path
- Dijkstra finds shortest path while accounting for different costs
- Both waste time exploring many directions that may not be worth it

## Searching for a Path in a Graph: Heuristic Search (greedy)



- Targeted expansion towards goal
- Driven by heuristic function
  - Example: distance to goal

## Searching for a Path in a Graph: Heuristic Search (greedy)



```

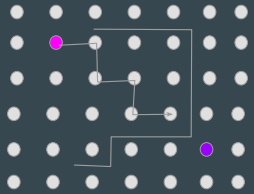
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    for next in graph.neighbors(current):
        if next not in came_from:
            # Greedy heuristic
            priority = distance(goal, next)
            frontier.put(next, priority)
            came_from[next] = current
    
```

## Searching for a Path in a Graph: Heuristic Search (greedy)



```

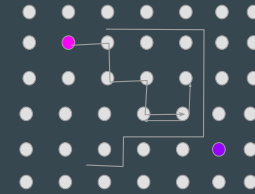
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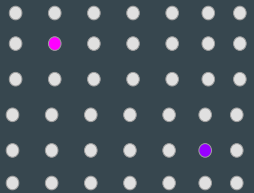
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```

- Effectiveness depends on heuristics
- There are No performance guarantees

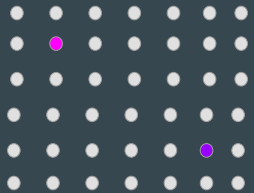
## Searching for a Path in a Graph: A\*



Best of both worlds

- Distance from home (Dijkstra)
- Distance from goal (Greedy)

## Searching for a Path in a Graph: A\*



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        new_cost = cost_so_far[current] + graph.cost(current, next)
        if next not in cost_so_far or new_cost < cost_so_far[next]:
            cost_so_far[next] = new_cost
            priority = new_cost + distance(goal, next)
            frontier.put(next, priority)
            came_from[next] = current
    
```

## Recalculation of paths

- World changes, path may not longer be optimal or be plain obsolete
- When
  - Every  $n$  steps (space or time)
  - When world change is detected
  - When landmarks are identified
  - When lost
  - When possible (extra time, CPU)
- What to recalculate
  - Full path
  - Partial path (closest) by splicing and stitching

## Key data structures in ROS for motion

```
# This represents a 2-D grid map
# Each cell represents the probability of occupancy
```

```
#Metadata for the map
MapMetaData info
```

```
# The map data, in row-major order, starting with (0,0). Occupancy
# probabilities are in the range [0,100]. Unknown is -1.
int8[] data
```

Occupancy Grid

## Key data structures in ROS for motion

Occupancy Grid for representing maps

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# The time at which the map was loaded
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# The map resolution [m/cell]
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# Map width [cells]
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uint32 height
# The origin of the map [m, m, rad]
# This is the real-world pose of the cell (0,0) in the map.
geometry_msgs/Pose origin
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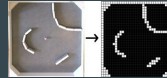
3D? Look at Octomaps  
<https://wiki.ros.org/octomap>



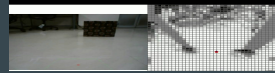
## Key data structures in ROS for motion

Occupancy Grid for representing maps

Cells containing 0,100



Cells containing range of probabilities between 0,100



## Key data structures in ROS for motion

Grid of cells -- same size cells, could be dispersed

```
# An array of cells in a 2D grid
float32 cell_width
float32 cell_height
geometry_msgs/Point[] cells
```



## Key data structures in ROS for motion

Grid of cells -- same size cells, could be dispersed

```
int array of cells in a 2D grid
float32 cell_width
float32 cell_height
geometry_msgs::msg::Cells cells
```

```
float This contains the position of a point in free space
float64 x
float64 y
float64 z
```

## Key data structures in ROS for motion

Path as a sequence of poses (waypoints + orientation)

```
int array of poses that represents a Path for a robot to follow
geometry_msgs::msg::Pose[] poses
```

```
float A representation of pose in free space, composed of position and orientation.
Point position
float64 x, y, z, Rotation w
Quaternion orientation
```

## Take Away

- Families of approaches to employ in tandem
  - Reactive
    - Local area and fast response
  - Model-based
    - Big picture and long paths
    - Build and searching graphs
  - ROS Support